

A Neural Path Integration Mechanism for Adaptive Vector Navigation in Autonomous Agents

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Abstract—Animals show remarkable capabilities in navigating their habitat in a fully autonomous and energy-efficient way. In many species, these capabilities rely on a process called path integration, which enables them to estimate their current location and to find their way back home after long-distance journeys. Path integration is achieved by integrating compass and odometric cues. Here we introduce a neural path integration mechanism that interacts with a neural locomotion control to simulate homing behavior and path integration-related behaviors observed in animals. The mechanism is applied to a simulated six-legged artificial agent. Input signals from an allothetic compass and odometry are sustained through leaky neural integrator circuits, which are then used to compute the home vector by local excitation-global inhibition interactions. The home vector is computed and represented in circular arrays of neurons, where compass directions are population-coded and linear displacements are rate-coded. The mechanism allows for robust homing behavior in the presence of external sensory noise. The emergent behavior of the controlled agent does not only show a robust solution for the problem of autonomous agent navigation, but it also reproduces various aspects of animal navigation. Finally, we discuss how the proposed path integration mechanism may be used as a scaffold for spatial learning in terms of vector navigation.

I. INTRODUCTION

Animals have evolved a remarkable variability of navigational behaviors to survive in complex dynamic environments, which is essential for finding vital locations, including food and shelter. While lower-order animals, such as nematodes, mainly navigate by directed walks using local cues, higher evolved animals (e.g., mammals, social insects) achieve more complex goal-directed navigation as well as path planning. These behaviors not only rely on sensory information, but also on internal representations from memory, and are often accompanied by learning mechanisms [1]. The ability of self-localization as well as acquiring and maintaining a metric reference frame is required to successfully navigate complex environments.

In order to maintain metric representations, animals integrate angular and linear ego-motion cues over time in order to maintain a vector representation of their current location with respect to their starting point, which is called the home vector. The underlying computation is termed Path Integration (PI) [2] or dead reckoning, and it is mainly used for homing,

i.e., returning back to the home. PI has been observed in many animals, including mammals [3], avians [4], fish [5], amphibians [6], arthropods [7], and presumably cephalopods [8]. It has been observed in different species of central place foragers, that PI plays a key role in navigation by providing local information to the agent when visual cues (e.g., landmarks) are abundant. There are also navigational behaviors mediated by PI. Searching patterns of desert ants have shown to be influenced by PI. Furthermore, the homing behavior after directionally biased outbound walks results in systematic errors, in which the animal misestimate the home position by a short distance located right in front of the actual nest. These errors have been observed in several species in vertebrates and invertebrates.

Neural substrates of PI have yet to be completely identified, but previous findings of neural representations of compass cues may provide essential information about how PI is achieved in neural systems [9], [10]. The firing pattern of certain neurons in the rat limbic system has been found to encode the animal's head orientations in the plane, independent of the animal's location in space [11]. These so-called head direction (HD) cells are derived from motor and vestibular sensory information by integrating head movements through space. A similar neural representation has been found in migratory insects. Cells in a central brain area, called central complex, encode azimuthal directions of the insect, which are derived from the polarization pattern induced by scattered sunlight [12]. Thus, neural substrates of allothetic compass cues have been found in both invertebrate and vertebrate species. These cues provide input signals for a potential PI mechanism based on the accumulation of azimuthal directions of the moving animal as previously proposed by [13].

Most models of PI have favored a particular coordinate system (Cartesian or polar) and reference frame (geo- or egocentric) to perform PI based on theoretical and biological arguments [14]. While some models [15], [16] include behavioral data from navigating animals in order to argue for their proposed PI method, other works [17], [18], [19] have applied neurobiologically realistic network models to investigate possible memory mechanisms for PI. Despite the wide variety of models, none of these models have been implemented on realistic embodied artificial agents in order to provide results of their model in the ecological context of

animals. Furthermore, possible links between PI and navigational capabilities, including spatial learning and memory, have largely been ignored.

This paper presents a neural mechanism for PI in embodied artificial agents. The mechanism is based on population encoding of heading directions in circular arrays, which are modulated by the walking speed of the agent. In order to perform PI, the head direction activity is accumulated by a self-recurrent connection. The final home vector representation is computed by local excitation-lateral inhibition connections, which projects accumulated heading directions onto the array of output neurons. Implemented on an embodied artificial agent, the mechanism not only allows for robust homing behavior in the presence of sensory noise, but also accounts for other PI-related behaviors observed in animals, including systematic errors, foraging and searching behaviors. Finally, we discuss our findings in order to relate the computational properties of our neural path mechanism to spatial learning and memory of vector representations.

II. CLOSED-LOOP CONTROL ARCHITECTURE FOR NAVIGATION

The neural path integration (PI) mechanism is implemented in a modular closed-loop architecture (Fig. 1a) embedded into a simulated, embodied artificial agent based on the six-legged walking robot *AMOS II* (Fig. 1b, [20]). The mechanism interacts with a CPG-based locomotion controller, which generates directed walking behavior based on the output signal of the PI mechanism. The CPG-based locomotion control has been presented in previous work [20]. Therefore, here we describe only its main function, while the neural PI mechanism is discussed in detail in the following section.

The CPG-based locomotion control consists of modular neural networks generating a variety of periodic patterns and coordinating all leg joints of the agent, thereby leading to a multitude of different behavioral patterns and insect-like leg movements. The resulting behaviors include omnidirectional walking and insect-like gaits [20], which can be controlled manually or autonomously driven by exteroceptive sensors, such as a camera [21], a laser scanner [22], or infrared sensors [20]. All neural networks in the CPG-based locomotion control are modeled using a discrete-time non-spiking neuron model with different activation functions (see [20] for details).

III. NEURAL PATH INTEGRATION MECHANISM

In this paper, we propose a neural path integration mechanism (Fig. 2) for embodied artificial agents. It consists of multiple circular arrays that act as processing layers, where the final layer's activity pattern represents the home vector. Our mechanism applies circular arrays of neurons with population-coded compass information and rate-coded linear displacements. Incoming signals are sustained through leaky neural integrator circuits, and they compute the home vector by local excitatory-lateral inhibitory interactions.

A. Sensory input

The sensory input to our mechanism consists of two sensors that perceive angular and linear motion cues for path integration. As in social insects, angular cues are derived

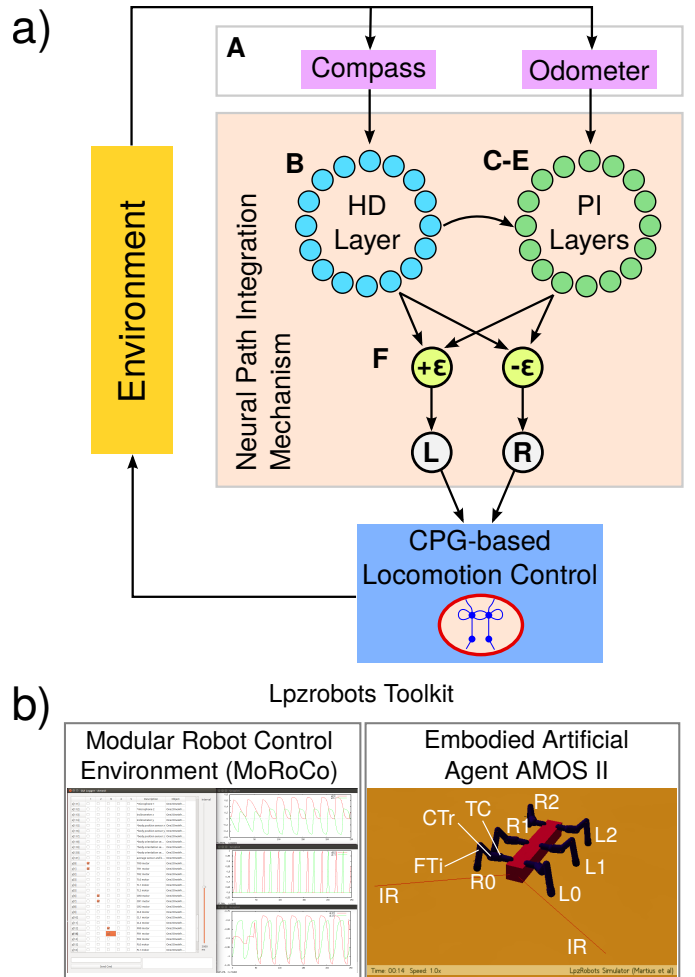


Fig. 1. Closed-loop control architecture and experimental setup for adaptive vector navigation in embodied artificial agents. (a) The architecture consists of the neural path integration (PI) mechanism and CPG-based locomotion control, which are implemented on a simulated, embodied artificial agent. The PI mechanism receives input from a compass sensor and odometry (A). Layers of circular arrays in the PI mechanism compute directional representations of the current head direction (B) and the home vector (C-E). The output signal (F) is the difference between current head direction and home vector direction, and controls the turning of the agent during locomotion. Note that the capital letters correspond to the subsections describing the Neural Path Integration Mechanism (Section III). The CPG-based locomotion control generates motor signals for directed walking behavior interacting with the environment. (b) *Lpzrobots* framework [23] containing the Modular Robot Control Environment and the simulated artificial agent based on the six-legged walking robot *AMOS II* [20]. The agent has six legs (R0, R1, R2, L0, L1, L2) and each leg has three joints: the thoraco-coxal (TC) joint enables forward and backward movements, the coxa-trochanteral (CTr) joint enables elevation and depression of the leg, and the femur-tibia (FTi) joint enables extension and exion of the tibia. The agent also contains a multitude of proprio- and exteroceptive sensors. In this paper, we use a compass sensor, a walking speed sensor, and infrared (IR) sensors.

from an allothetic compass sensor which measures the angle ϕ of the agent's orientation. In insects, this information is derived from the combination of sun- and skylight compass information [10]. Odometry is provided by a speed sensor measuring the walking speed s of the agent. For the hexapod robot, the walking speed is computed by accumulating steps and averaging over a certain time window. These step counting signals are derived from the motor signals.

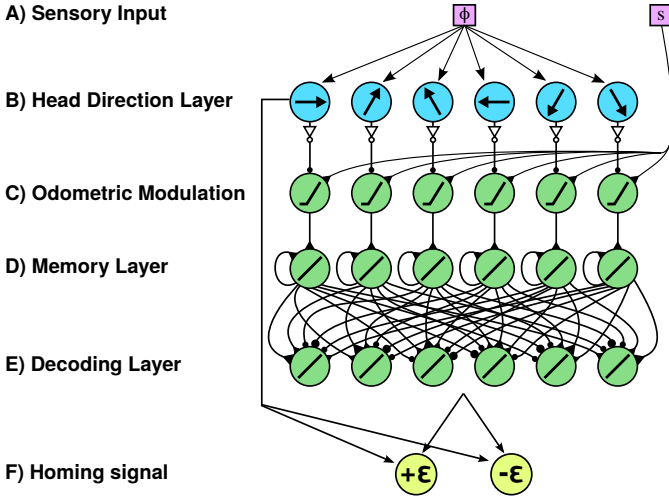


Fig. 2. Processing layers of the Neural Path Integration Mechanism. Each neuron encodes a particular preferred direction enclosing the full range of 2π . Note that the figure depicts only six neurons for simplicity. A) Sensory input from a compass sensor (ϕ) and walking speed sensor (s) is provided to the mechanism. B) Neurons in the head direction layer encodes the sensory input from a compass sensor using a cosine response function. C) Head direction signals are modulated by the walking speed of the agent. D) The memory layer accumulates the signals by a self-recurrent connection. E) A cosine weight kernel decodes the accumulated directions to compute the output activity representing the home vector. F) The difference in home vector direction and current head direction is used to compute the homing signal.

$$\phi \in [0, 2\pi) \quad (1)$$

$$s \in [0, 1] \quad (2)$$

B. Head direction layer

The first layer of the neural network model consists of head direction (HD) sensitive cells with activation functions

$$x_i^{\text{HD}}(\phi(t)) = \cos(\phi(t) - \phi_i), \quad (3)$$

$$\phi_i = \frac{2\pi i}{N}, \quad i \in [0, N - 1], \quad (4)$$

where the compass signal is encoded by a cosine response function with N preferred directions $\phi_i \in [0, 2\pi)$. The coarse encoding by the cosine ensures high resolution and optimizes information transfer [24].

C. Odometric modulation of head direction signals

The second layer acts as a gating mechanism (G), that modulates the neural activity using the walking speed signal $s \in [0, 1]$. Thus, it encodes in its activity, the travelled distances of the agent. The gating layer units decrease the HD activities by a constant bias of 1, so that the maximum activity is equal to zero. A positive speed increases the signal linearly. The gating activity is defined as follows:

$$x_i^{\text{G}}(t) = f(\delta_{ij} x_j^{\text{HD}}(t) - 1 + s), \quad (5)$$

$$f(x) = \max(0, x) \quad (6)$$

where δ_{ij} is the Kronecker delta, i.e., the first and second layer are connected one-to-one, and f is the linear rectifier transfer function. The linear rectifier function only transmits

positive signals. Examples of different speed-modulated gating activities are shown in Fig. 3.

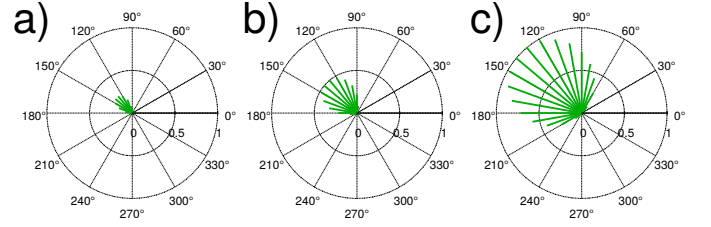


Fig. 3. Example of speed modulation using a gating mechanism with 36 neurons. The agent's heading direction is $\phi = 135^\circ$ and the speed s is set to (a) 0.25, (b) 0.5, and (c) 1.0.

D. Memory layer

The third layer is the so-called memory layer (M), where the speed-modulated head direction activations are temporally accumulated through a self-excitatory connection:

$$x_i^{\text{M}}(t) = \delta_{ij} x_j^{\text{G}}(t) + (1 - \lambda) x_i^{\text{M}}(t - 1) \quad (7)$$

where λ is a positive constant defined as the integrator leaking rate, which indicates the loss of information over time.

E. Decoding layer

Finally, the fourth layer decodes the activations from the memory layer to generate a vectorial representation of the home vector which is the output of the mechanism (also called PI state):

$$x_i^{\text{PI}}(t) = w_{ij} x_j^{\text{M}}(t) \quad (8)$$

$$w_{ij} = \cos(\phi_i - \phi_j) \quad (9)$$

where w_{ij} is a cosine kernel that decomposes the projections of the memory layer activations. The resulting home vector is encoded by the average position of maximum firing in the array (angle θ) and the sum of all firing rates of the array (length l). See Fig. 4 for example output activities of the decoding layer neurons.

F. Homing signal

In order to apply the home vector information for homing behavior, the vector simply needs to be rotated by 180° . The angular error between the current heading direction ϕ and the current inverted home vector direction $\theta - \pi$ is used for steering the agent towards home. The agent applies homing by error compensation, which defines the motor command

$$\epsilon = \sin(\theta - \phi - \pi). \quad (10)$$

As a result, positive and negative errors induce right ($\epsilon < 0$) and left turns ($\epsilon > 0$), respectively, reducing the net error at each step.

IV. SIMULATION RESULTS

In order to evaluate our path integration mechanism proposed in the previous section, we carry out a series of experiments that test the capabilities and robustness of the mechanism under behaviorally relevant conditions. We provide

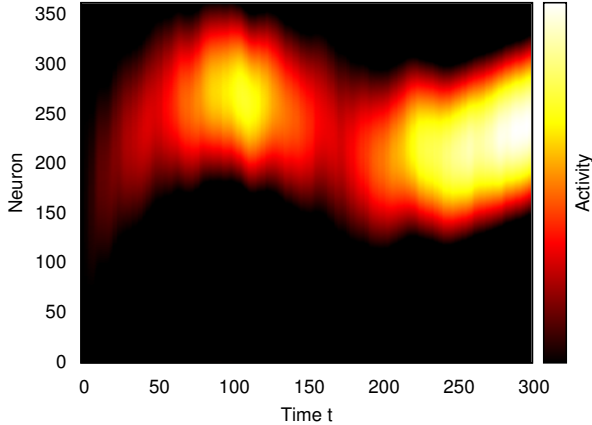


Fig. 4. Neural activities of the PI neurons for an example (random) walk.

results using two different experimental platforms. A two-dimensional point process simulation is used for the main analysis of our control mechanisms. It can efficiently carry out large number of experimental trials, which is required to evaluate our models in a time-efficient manner. The second platform is an embodied artificial agent simulated using the robotic simulation framework called *Lpzrobots* [23]. The biomechanics of the agent are based on the six-legged walking robot *AMOS II* [20]. Its biomechanical similarities with insects enables an appropriate embodied model for the study of insect-inspired navigation control.

We test the proposed path integration mechanism in two different experimental setups. First, we let the agent run outbound from an initial home position with two segments of fixed orientations. These runs are referred to as L-shaped runs, where the angle α defines the angle between the two segments. L-shaped runs are applied under different noise levels to evaluate robustness to noise, and to account for the observed systematic errors observed during homing. Furthermore, we tested our mechanism in a second experimental setup, in which the agent runs outbound in random orientations. In animals, this foraging behavior serves as an exploration mechanism to find goals in the environment when no indicating stimulus is available.

A. Homing behavior

Here we evaluate the capabilities of our mechanism for outbound runs with fixed orientations of the agent. Each run consists of two straight legs in a prefixed direction of $\alpha_1 = \frac{3\pi}{2}$ and $\alpha_2 = \pi$, respectively. Both path segments have the same length of 5. Fig. 5 shows the performance of homing behavior for different levels of sensory noise added to the path integration mechanism. For noise levels up to 5%, the path integration mechanism provides reliable estimates of the home position. The estimated home is indicated by the looping behavior of the agent, which results from the sine error compensation for homing [14].

In Fig. 6, we show the density map of the estimated home position using our path integration mechanism with respect to

different sensory noise levels. Increasing noise levels leads to larger uncertainty in determining the current position of the agent. For noise levels up to 5%, the agent estimates the home position with high probability (indicated by the red color) near to actual home (green circle). This demonstrates the noise robustness of our mechanism, which offers useful applications for physical mobile robots dealing with a high degree of noise in sensing and actuation.

Figure 7 shows that the PI mechanism is able to effectively deal with deviations in the homing direction. This is tested by placing an obstacle in the way of the initial homing trajectory. As the agent approaches the obstacle, the infrared (IR) sensors attached at the front of the agent detect the obstacle and drive obstacle avoidance behavior. After obstacle avoidance, the agent has deviated from the initial homing trajectory and the resulting home vector direction has changed. However, since the PI mechanism runs continuously, it can compensate for such deviations and allow the agent to reach the desired home position. A similar compensation during PI-driven homing behavior has also been observed in animal experiments [3], [10]. The video clip of this experiment can be seen at www.manoonpong.com/IJCNN15/S1.mp4.

B. Systematic errors in homing experiments

Many invertebrate and vertebrate species exhibit systematic error during homing after following an L-shaped outward journey (reviewed in [3]). Müller and Wehner [15] have examined these errors in desert ants by measuring the angular deviation with respect to the angle of the L-shaped course. In order to show that our mechanism is able to reproduce these errors, we fit our model against the desert ant data from [15], [25] using the leaking rate λ (Eq. 7) of the path integration memory layer as control variable (see Fig. 8). Using a leaking rate of $\lambda \approx 0.0075$ resulted in angular errors most consistent with behavioral data. The idea of leaky integration producing systematic errors has also been previously proposed by [26], [14]. Thus, here our mechanism not only provides a robust solution for homing, but it also reproduces behavioral aspects observed in animals.

C. Foraging experiments

In order to find resources, animals explore their environments in seemingly randomized patterns. In order to test that our mechanism can also perform under these conditions, we carried out homing experiments after random outbound runs of the agent. Random exploration was achieved by using a gaussian distribution with zero mean and standard deviation 0.15 for the turning rate $\frac{d\phi}{dt}$. We tested the performance in terms of angular deviation from actual home with respect to two important model parameters, namely, sensory noise and number of neurons. The experiments were averaged over 1000 trials. Fig. 9 shows the effect of different degrees of sensory noise on the performance of path integration for a fixed number of 360 neurons per layer. For noise levels up to 5% (equal to 18°), the observed mean angular error is below 5° indicating the robustness of our path integration mechanism. Here, the coarse population coding of heading directions directly results in reduction of noise from incoming sensory signals.

In Fig. 10, we varied the number of neurons in the circular arrays of the path integration mechanism keeping the sensory

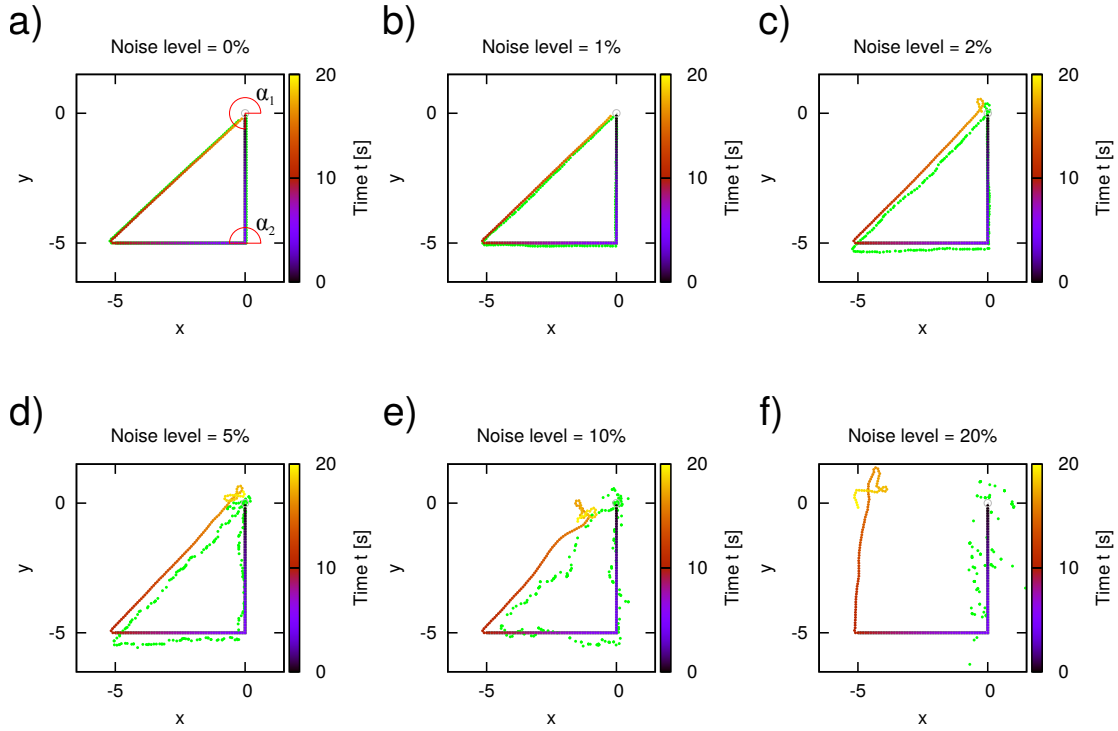


Fig. 5. L-shaped outbound runs and homing behavior using a path integration mechanism for six different levels of sensory noise: (a) 0%, (b) 1%, (c) 2%, (d) 5%, (e) 10%, and (f) 20%. The multi-colored trajectory is the agent's position over time. The green-colored trajectory indicates the estimated positions retrieved from the path integration mechanism. The actual prefixed directions α_1, α_2 are indicated in (a) for ease of understanding.

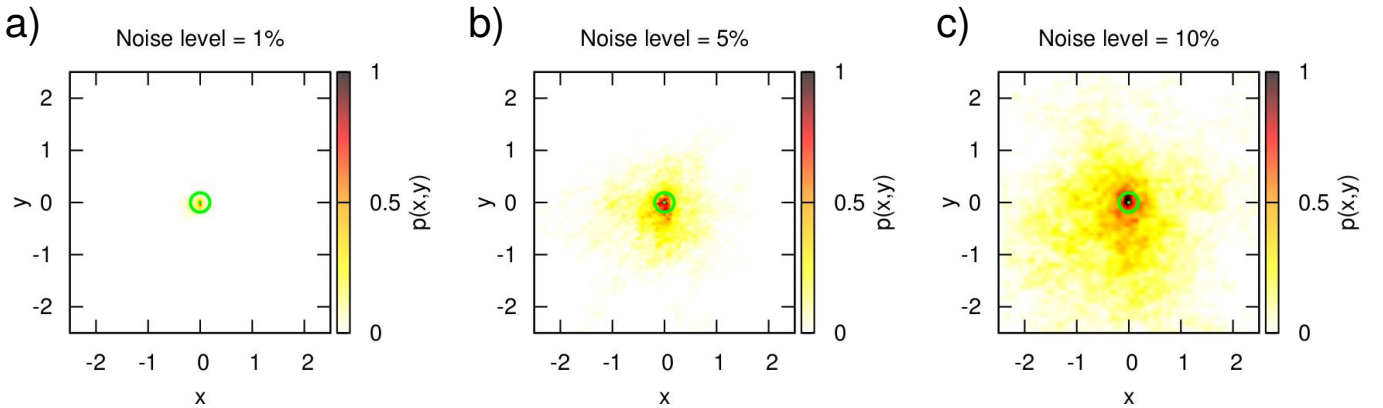


Fig. 6. Density maps of estimated home positions using a path integration mechanism for three different levels of sensory noise: (a) 1%, (b) 5%, (c) 10%. The green circle corresponds to the home position with radius $r_{home} < 0.2$.

noise level fixed at 5%. The low and relatively stable mean angular error clearly indicates that the mechanism can produce fairly accurate home vector estimates even with 100 neurons. This is again mainly due to the coarse coding of heading directions.

V. CONCLUSION AND DISCUSSION

In this paper, we presented a neuro-inspired mechanism capable of performing path integration. The mechanism is fed by inputs from an allothetic compass and an odometer

providing sensory modalities similar to insects. The home vector is computed and represented in circular arrays of neurons where heading angles are population-coded and linear displacements are rate-coded. This neural representation of spatial knowledge is also found in insects [12]. The coarse encoding of orientation using cosine responses was previously applied by other models [18], [19]. Contrary to these works, our model integrates the walking speed signal as an additive factor, instead of a multiplicative factor. Temporal integration of the speed-modulated head direction signals is achieved by

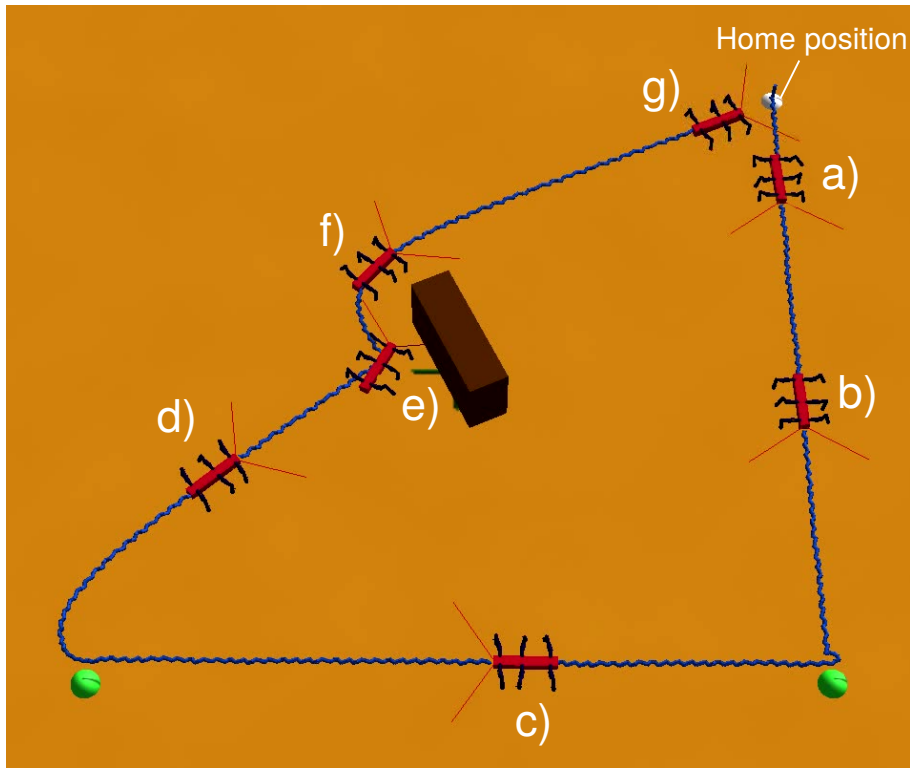


Fig. 7. Multiple, overlaid frames of PI-mediated homing and obstacle avoidance in an embodied artificial agent. (a) The agent runs out from its home position (white-colored sphere) and (b) keeps its head direction at a fixed angle, (c) before turning right by 90° . After the L-shaped outward run, (d) the agent performs homing behavior provided by the neural PI mechanism. (e) An obstacle is placed in the way of the homing trajectory and the agent performs obstacle avoidance driven by its IR sensors. (f) The PI mechanism runs continuously and can therefore adjust the homing direction compensating the detour of avoiding the obstacle. See www.manoonpong.com/IJCNN15/S1.mp4 for the video clip of this experiment.

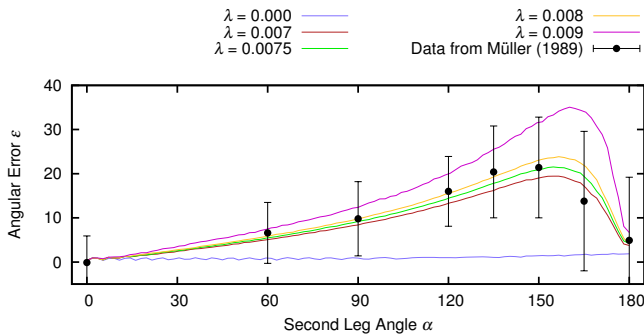


Fig. 8. Systematic errors of desert ant homing are reproduced by leaky integration of path segments. The second leg angle α is varied in 2.5° intervals for the simulation results. A leaking rate of $\lambda \approx 0.0075$ is used to fit the behavioral data [15], [25].

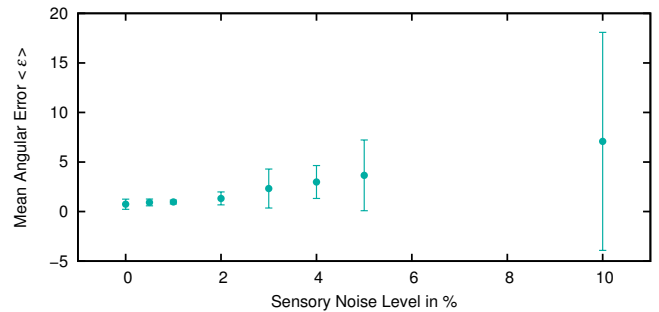


Fig. 9. Mean angular errors (\pm S.D.) in path integration with respect to sensory noise levels averaged over 1000 trials (fixed number of 360 neurons per layer).

leaky neural integrator circuits, which are modeled by simple self-recurrent loops. Biologically these recurrent connections can be interpreted as positive feedback within a group of neurons with the same preferred direction. From a theoretical point of view, we apply this simplified mechanism to avoid random drifts, which are observed in more complex attractor networks [27] proposed in previous path integration models [17], [28]. Finally, the home vector is computed by local excitatory-lateral inhibitory interactions using a cosine weight kernel. This has been previously applied in [29] and it provides the mathematically correct decomposition of the projections

of each direction. Furthermore, in contrast to many existing works, we analyzed the effects of sensory noise on the accuracy of our path integration mechanism. We show that the noise robustness of our path integration mechanism allows the implementation on realistic embodied artificial agents for testing it in the ecological context of animals.

Path integration is a fundamental, but yet challenging computation done by the animal nervous systems. It is employed by central place foragers, such as social insects, to locate its current position with respect to its nest. In rodents and other vertebrates, path integration is useful when visual information is abundant. The underlying neural basis of path integration

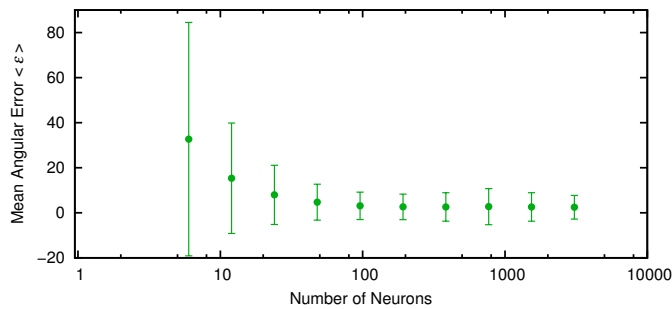


Fig. 10. Mean angular errors (\pm S.D.) in path integration with respect to number of neurons (axis in logarithmic scale) averaged over 1000 trials (sensory noise level fixed at 5%).

has not been fully understood neither in invertebrates, nor in vertebrates. However, the findings of the main neural pathways, as well as the neural coding of sensory cues required for path integration have shed light onto how animals might process angular and linear self-motion cues [10], [11], [12].

Application of our proposed neural mechanism clearly demonstrates the ability to successfully navigate a two-dimensional point agent as well as the simulated physical robot back to its home position. The leakage parameter λ provided a single free parameter to successfully fit our model to behavioral data from desert ants [15]. Other models also provided this evidence [15], [16], [17], however in our model, memory decay is seen as the reason for the observed systematic errors [14], [26]. The extensive analysis of multiple simulations with random foraging revealed that path integration accumulates errors over time. In the presence of noise, the path integration mechanism is still capable of producing accurate estimates of the home vector. We would like to emphasize the application of our mechanism to an embodied legged agent. This experimental platform provides a valid biomechanical model of insects for modelling spatial behavior, such as path integration and homing under similar conditions as in nature. To our knowledge, existing models of PI mechanisms have hitherto not been tested on legged robots. Although we have demonstrated robust homing behavior using a simulated robot, in future work, we plan to extend this to a physical walking robot *AMOS II* [20]. We consider the use of an inertial measurement unit (IMU), with an integrated accelerometer, gyroscope, and magnetometer, serving as compass input to our path integration mechanism. Robust sensory preprocessing using reservoir computing [30] allows for compensating noisy or missing sensory information from the IMU. Future experiments will also involve testing our path integration mechanism in more complex environments with irregular terrain. In such a terrain, accurate path integration requires the estimation of ground-projected paths [31]. This is possible by including the agent's inclination as a modulatory signal to the gating layer of our mechanism. Navigating in complex, real-world environments creates the need for robust and adaptive locomotor behaviors, which have been previously presented on *AMOS II* [32], [33]. Furthermore, we will apply an acoustic source and bio-inspired auditory sensors for the localization of goals.

Besides homing being an obvious behavior mediated by path integration, it can also act as a necessary scaffold for

spatial learning [34]. By providing a metric representation of the space, the path integrator state may be associated with a reward received at certain locations, such as feeding sites, and help the learning of visually-guided landmark responses. A recent study [35] suggest a shared representation of goal direction and head direction using multivoxel pattern analysis of fMRI measurements, which supports the hypothesis that representations of goal directions may be learned by storing head direction-accumulated path integration states. In future work, we will test this hypothesis by combining the neural PI mechanism with a reward-modulated associative learning rule for spatial learning of goal locations. The learned representations of goals are global vectors based on the PI state experienced at the goal location. Furthermore, such mechanisms can be linked to complex adaptive vector navigation including decision making [36] and route learning [37] based on local vectors.

ACKNOWLEDGMENT

This research was supported by the Emmy Noether Program (DFG, MA4464/3-1), and the Federal Ministry of Education and Research (BMBF) by a grant to the Bernstein Center for Computational Neuroscience II Göttingen (01GQ1005A, project D1).

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